# Nucleon Electro-Magnetic Form Factors

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# Introduction

- Form Factor

  response of system to momentum transfer Q, often normalized to that of point-like system

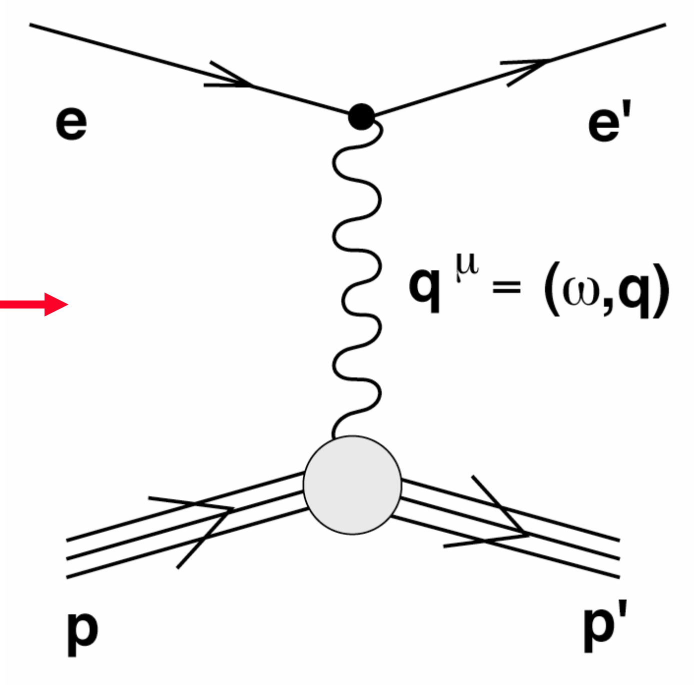
  Examples:

  →scattering of photons by bound atoms

  →nuclear beta decay

  →X-ray scattering from crystal

  →electron scattering off nucleon



# Nucleon Electro-Magnetic Form Factors

- Fundamental ingredients in "Classical" nuclear theory
- A testing ground for theories constructing nucleons from quarks and gluons
  - spatial distribution of charge, magnetization

  wavelength of probe can be tuned by selecting momentum transfer Q:

  < 0.1 $GeV^2$ integral quantities (charge radius,…)

  0.1-10 $GeV^2$ internal structure of nucleon

  > 20 $GeV^2$ pQCD scaling

  Caveat: If Q is several times the nucleon mass (~Compton wavelength), dynamical effects due to relativistic boosts are introduced, making physical interpretation more difficult

- Additional insights can be gained from the measurement of the form factors of nucleons embedded in the nuclear medium
  - implications for binding, equation of state, EMC…
  - precursor to QGP

# Campaigns and Performance Measures

How are nucleons made from quarks and gluons?

The distribution of u, d, and s quarks in the hadrons

(the spatial structure of charge and magnetization in the nucleons is an essential ingredient for conventional nuclear physics; the flavor decomposition of these form factors will provide new insights and a stringent testing ground for QCD-based theories of the nucleon)

DOE Performance Measures

2010 Determine the four electromagnetic form factors of the nucleon to a momentum-transfer squared, $Q^2$, of 3.5 $GeV^2$ and separate the electroweak form factors into contributions from the u,d and s-quarks for $Q^2 < 1$ $GeV^2$

# Formalism

Sachs Charge and Magnetization Form Factors $G_E$ and $G_M$

$$\frac{d\sigma}{d\Omega}(E,\theta) = \sigma_M \left[ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2(\theta/2) \right]$$

$$\sigma_M = \frac{\alpha^2 E' \cos^2(\theta/2)}{4E^3 \sin^4(\theta/2)}$$

with E (E') incoming (outgoing) energy, $\theta$ scattering angle,
$\kappa$ anomalous magnetic moment

In the Breit (centre-of-mass) frame the Sachs FF can be written as the Fourier transforms of the charge and magnetization radial density distributions

$G_E$ and $G_M$ are often alternatively expressed in the Dirac (non-spin-flip) $F_1$ and Pauli (spin-flip) $F_2$ Form Factors

$$F_1 = G_E + \tau G_M \qquad F_2 = \frac{G_M - G_E}{\kappa(1+\tau)} \qquad \tau = \frac{Q^2}{4M^2}$$

# The Pre-JLab Era

- Stern (1932) measured the proton magnetic moment $\mu_p \sim 2.5\ \mu_{Dirac}$
  indicating that the proton was not a point-like particle
- Hofstadter (1950's) provided the first measurement of the proton's
  radius through elastic electron scattering
- Subsequent data ($\lesssim$ 1993) were based on:
  Rosenbluth separation for proton,
  severely limiting the accuracy for $G_E^p$ at $Q^2 > 1$ GeV$^2$

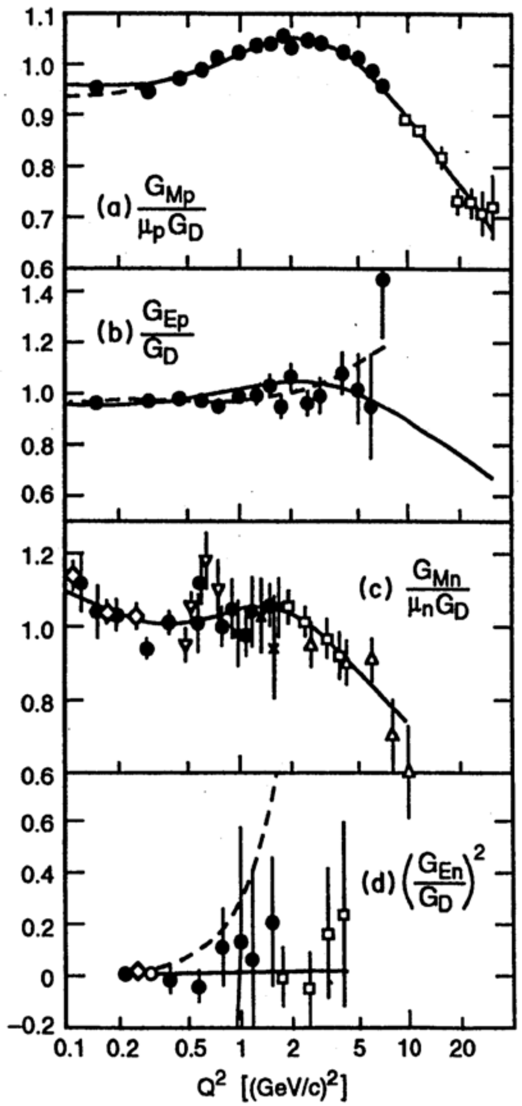
- Early interpretation based on Vector-Meson Dominance

- Good description with phenomenological dipole form factor:

$$\boxed{G_D = \left\{ \frac{\Lambda^2}{\Lambda^2 + Q^2} \right\}^2 \quad \textbf{with} \ \ \Lambda = 0.84\,GeV}$$

corresponding to $\rho$ (770 MeV) and $\omega$ (782 MeV) meson resonances
in timelike region and to exponential distribution in coordinate space

# Global Analysis




P. Bosted *et al.*
PRC 51, 409 (1995)

$$G_E^p = G_M^p = \left(1 + \sum_{i=1}^{5} a_i Q^i\right)$$

$$G_M^n = \left(1 + \sum_{i=1}^{4} b_i Q^i\right); \quad G_E^n = 0$$

Three form factors very similar

$G_E^n$ zero within errors -> accurate data on $G_E^n$ early goal of JLab

First JLab $G_E^p$ proposal rated $B^+$!

# Modern Era

Akhiezer et al., Sov. Phys. JETP 6 (1958) 588 and
Arnold, Carlson and Gross, PR C 23 (1981) 363
showed that:

accuracy of form-factor measurements can be significantly improved by measuring an interference term $G_E G_M$ through the beam helicity asymmetry with a polarized target or with recoil polarimetry

Had to wait over 30 years for development of

- Polarized beam with
  high intensity (~100 μA) and high polarization (>70 %)
  (strained GaAs, high-power diode/Ti-Sapphire lasers)
- Beam polarimeters with 1-3 % absolute accuracy
- Polarized targets with a high polarization or
- Ejectile polarimeters with large analyzing powers

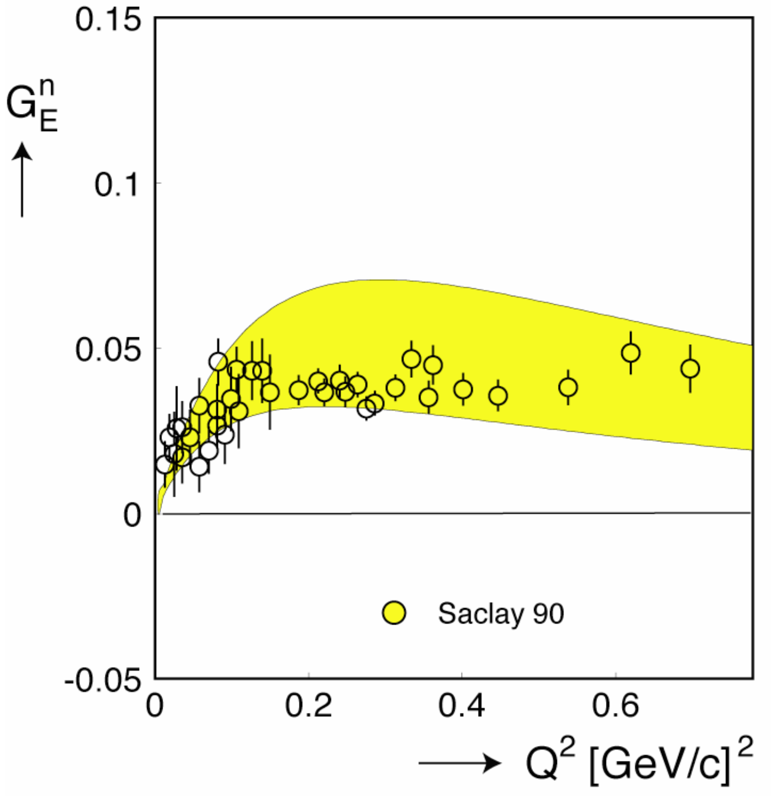
# Pre-Jlab Measurements of $G_E^n$

No free neutron target available, early experiments used deuteron
Large systematic errors caused by subtraction of proton contribution

Elastic e-d scattering
(Platchkov, Saclay)

$$\frac{d\sigma}{d\Omega} \propto \{A + B\tan^2(\theta_{e/2})\} \propto (G_E^p + G_E^n)^2 \int [u^2(r) + w^2(r)]\, j_0(\frac{Qr}{2})\, dr + ....$$

Yellow band represents range of $G_E^n$-values resulting from the use of different NN-potentials



# Double Polarization Experiments to Measure $G^n_E$

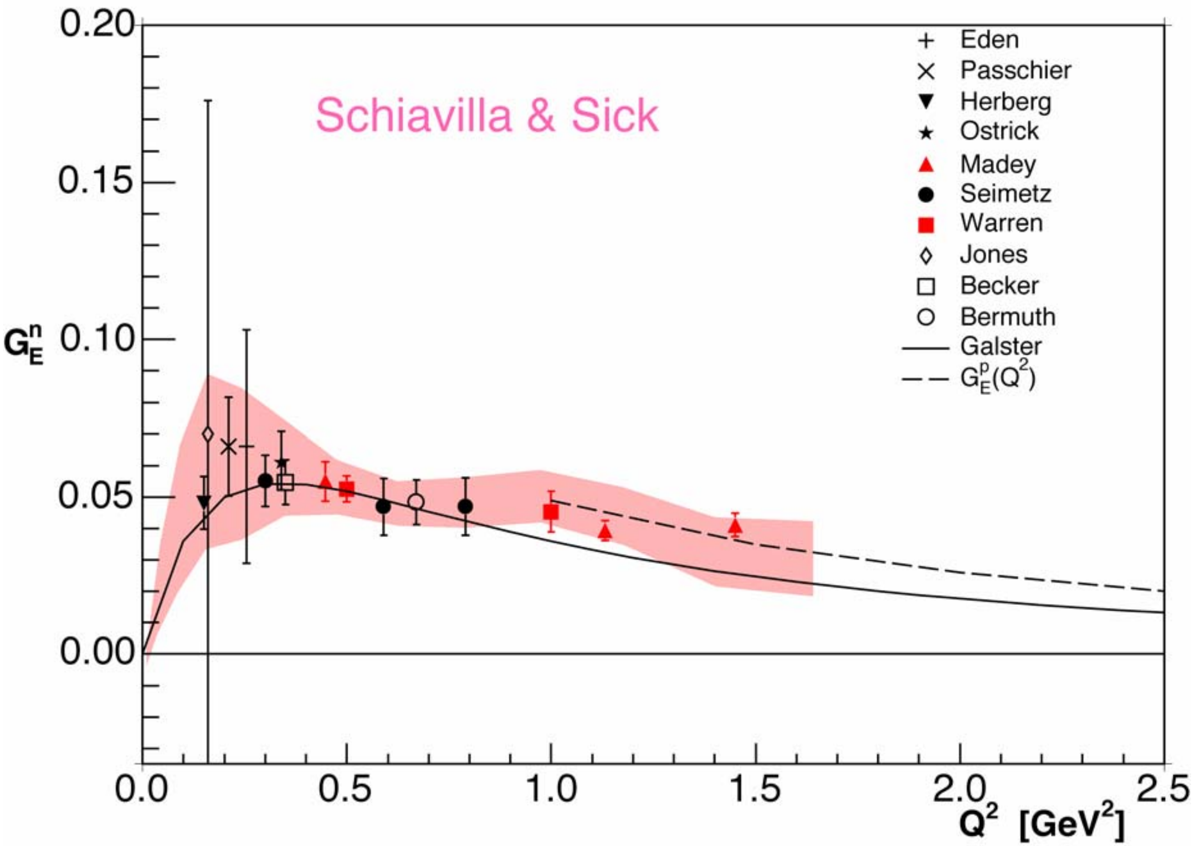
- Study the (e,e'n) reaction from a polarized $ND_3$ target
  limitations: low current (~80 nA) on target
  deuteron polarization (~25 %)

- Study the (e,e'n) reaction from a $LD_2$ target and measure the neutron polarization with a polarimeter
  limitations: Figure of Merit of polarimeter

- Study the (e,e'n) reaction from a polarized $^3$He target
  limitations: current on target (12 $\mu$A)
  target polarization (40 %)
  nuclear medium corrections

$$\boxed{\frac{G^n_E}{G^n_M} = \frac{A_\wedge}{A_{||}} \sqrt{t + t(1+t)\tan^2(q/2)}}$$

# Neutron Electric Form Factor $G_E^n$




Galster:
a parametrization fitted to old (<1971) data set of very limited quality

For $Q^2 > 1$ GeV$^2$ data hint that $G_E^n$ has similar $Q^2$-behaviour as $G_E^p$

# Measuring $G^n_M$

Old method: quasi-elastic scattering from $^2H$
large systematic errors due to subtraction of proton contribution

$$R_D = \frac{\dfrac{d^3\sigma(eD \Rightarrow e'n(p))}{dE'\,d\Omega_{e'}d\Omega_n}}{\dfrac{d^3\sigma(eD \Rightarrow e'p(n))}{dE'\,d\Omega_{e'}d\Omega_p}}$$

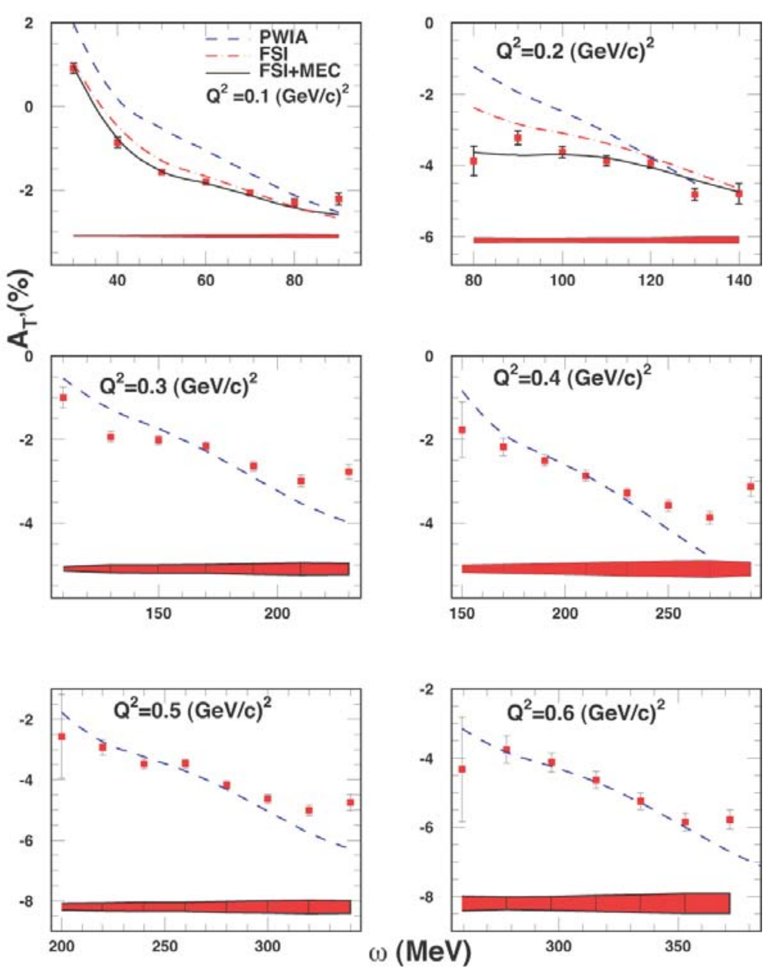
- Measure (en)/(ep) ratio
  - Luminosities cancel
  - Determine neutron detector efficiency
    - On-line through $e+p \to e'+\pi^+(+n)$ reaction (CLAS)
    - Off-line with neutron beam (Mainz)

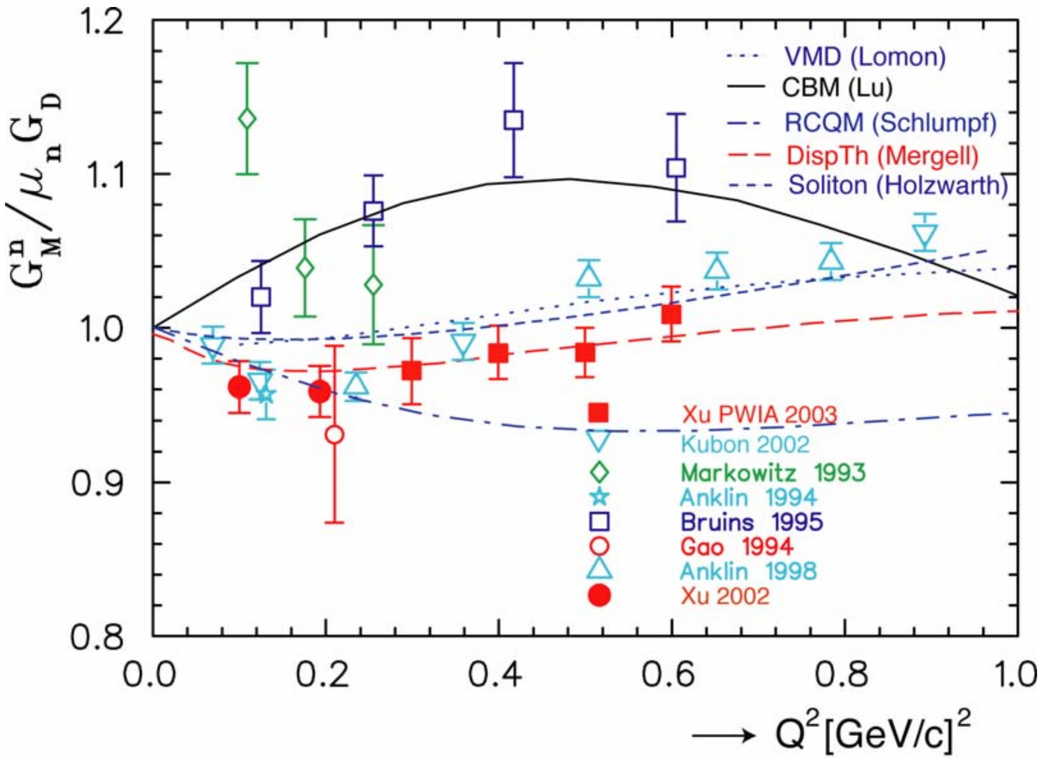
- Measure inclusive quasi-elastic scattering off polarized $^3He$ (Hall A)

$$A = \frac{-\left(\cos\theta^* v_{T'}R_{T'} + 2\sin\theta^*\cos\varphi^* v_{TL'}R_{TL'}\right)}{v_L R_L + v_T R_T}$$

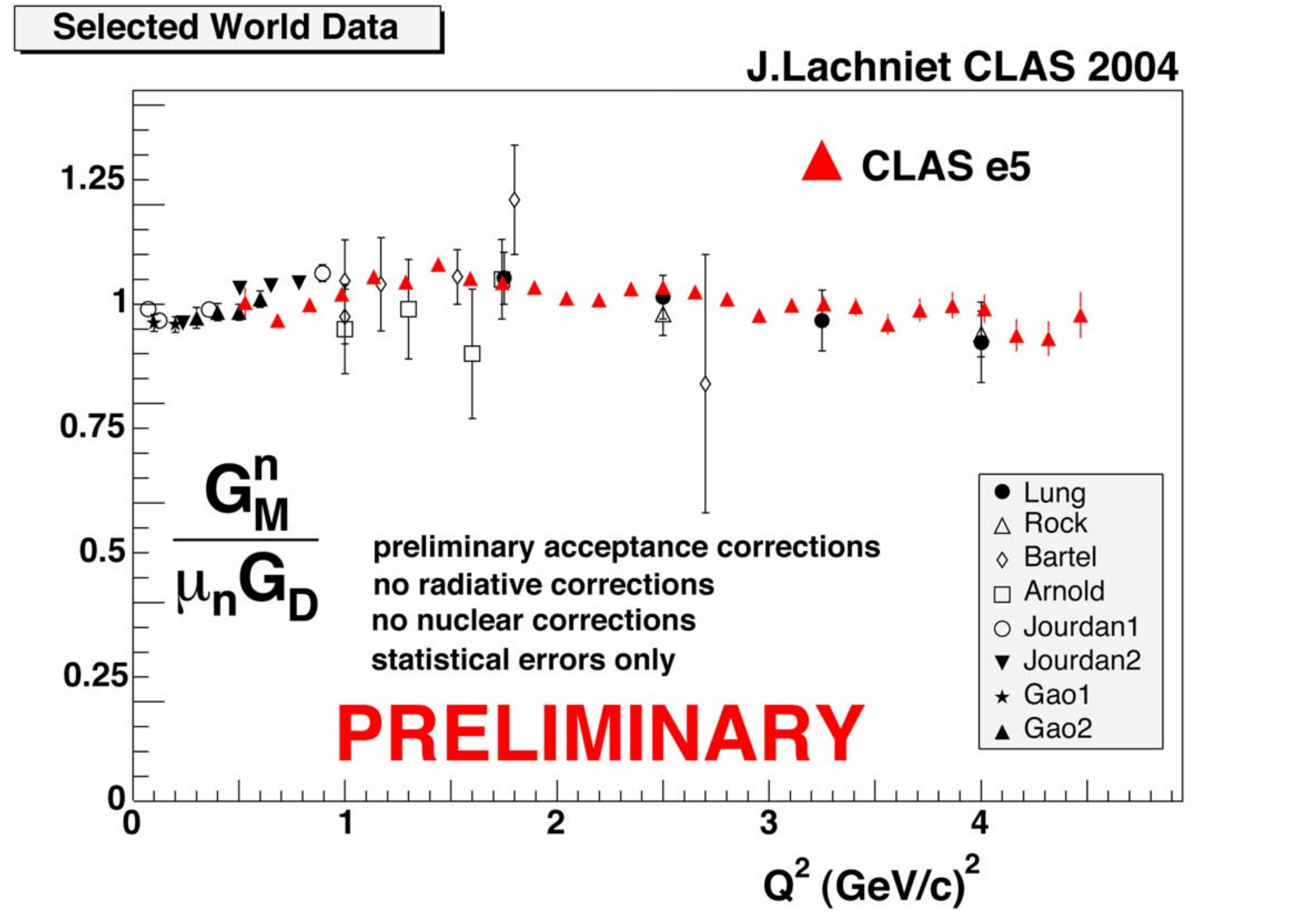
$R_{T'}$ directly sensitive to $(G_M^n)^2$

# Measurement of $G^n_M$ at low $Q^2$



# Preliminary $G^n_M$ Results from CLAS





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# Early Measurements of $G_E^p$

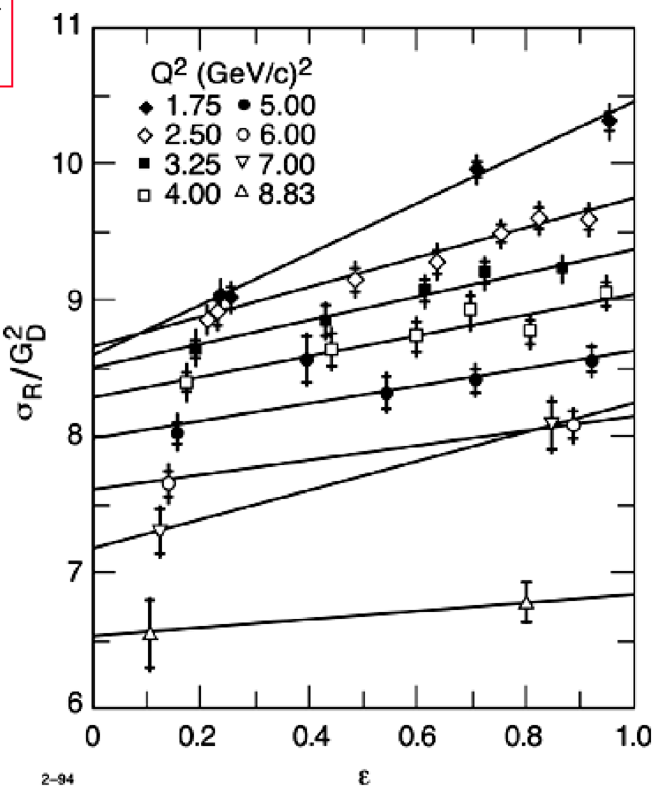
- relied on Rosenbluth separation
- measure $d\sigma/d\Omega$ at constant $Q^2$
- $G_E^p$ inversely weighted with $Q^2$, increasing the systematic error above $Q^2 \sim 1$ GeV$^2$

$$\sigma_R(Q^2,\varepsilon) = \varepsilon\left(1+\frac{1}{\tau}\right)\frac{E}{E'}\frac{\sigma(E,\theta)}{\sigma_{Mott}} = \{G_M^p(Q^2)\}^2 + \frac{\varepsilon}{\tau}\{G_E^p(Q^2)\}^2$$

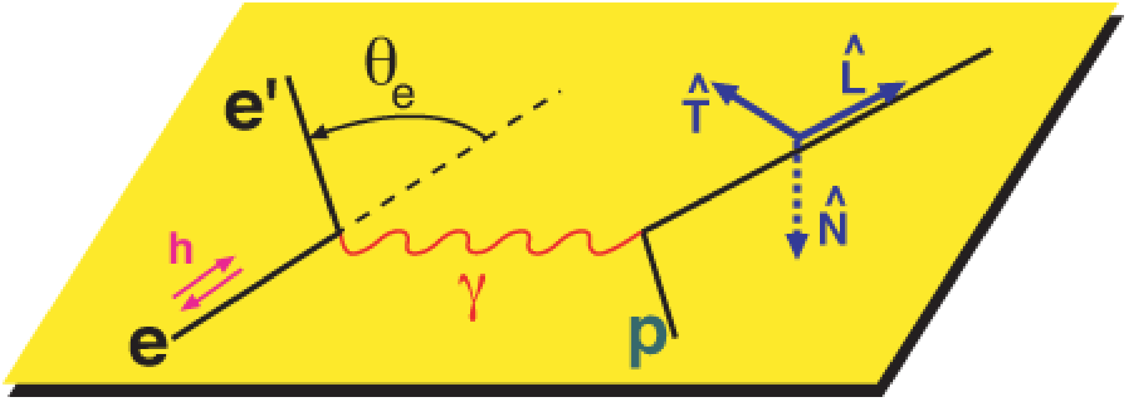
$$Q^2 = 4EE'\sin^2(q/2)$$

$$\varepsilon = \frac{1}{1+2(1+\tau)\tan^2(\theta/2)}$$

At 6 GeV$^2$ $\sigma_R$ changes by only 8% from $\varepsilon$=0 to $\varepsilon$=1 if $G_E^p = G_M^p/\mu_p$
Hence, measurement of $G_e^p$ with 10% accuracy requires 1.6% cross-section measurements over a large range of electron energies



# Spin Transfer Reaction $^1H(\vec{e},e'\vec{p})$




$$P_n = 0$$

$$\pm hP_t = \mp h\,2\sqrt{\tau(1+\tau)}\,G_E^p G_M^p \tan(\theta_e/2)/I_0$$

$$\pm hP_l = \pm h(E_e + E_{e'})(G_M^p)^2\sqrt{\tau(1+\tau)}\tan^2(\theta_e/2)/M/I_0$$

$$I_0 = \{G_E^p(Q^2)\}^2 + \tau\{G_M^p(Q^2)\}^2\left[1 + 2(1+\tau)\tan^2(\theta_e/2)\right]$$

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{E_e + E_{e'}}{2M}\tan(\theta_e/2)$$

No error contributions from

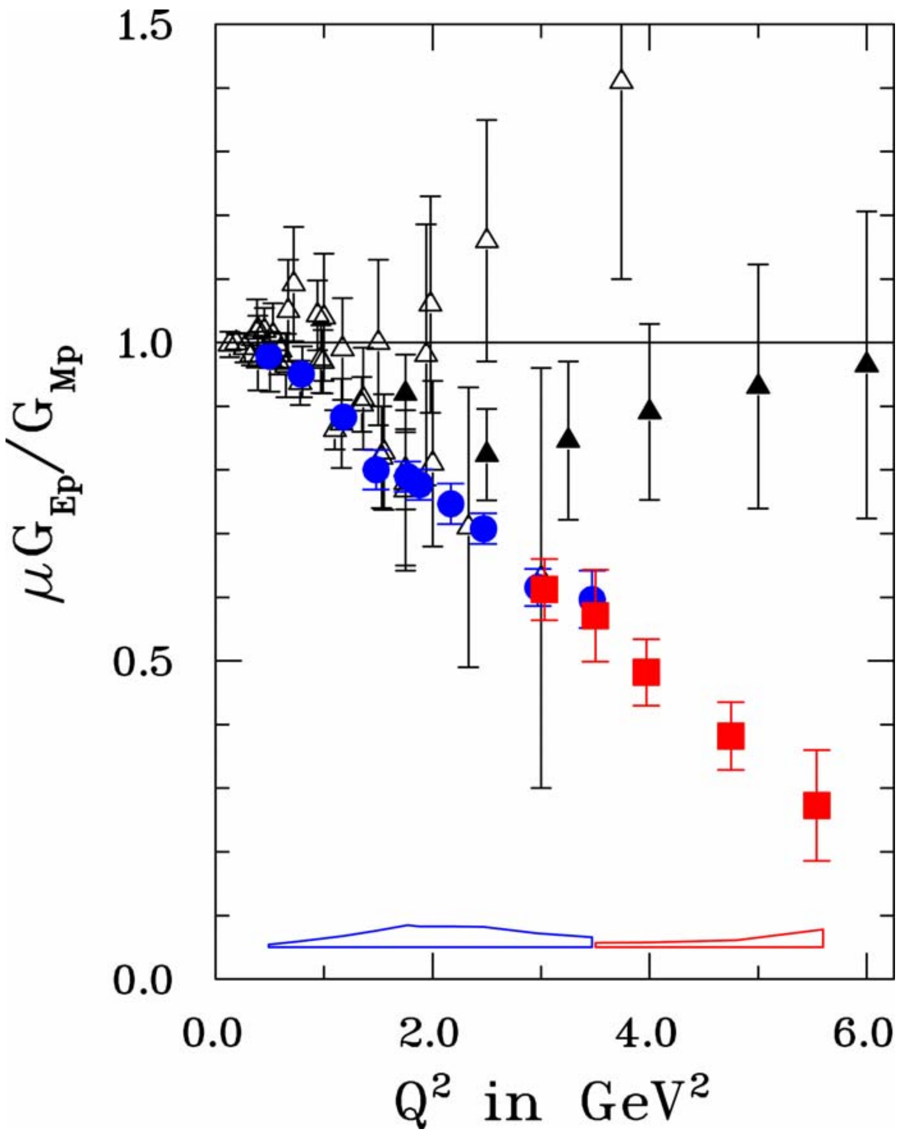
- analyzing power
- beam polarimetry

# JLab Polarization-Transfer Data

- E93-027 PRL 84, 1398 (2000)
  Used both HRS in Hall A with FPP
- E99-007 PRL 88, 092301 (2002)
  used Pb-glass calorimeter for electron detection to match proton HRS acceptance
- Reanalysis of E93-027 (Pentchev)
  Using corrected HRS properties

→ Clear discrepancy between polarization transfer and Rosenbluth data

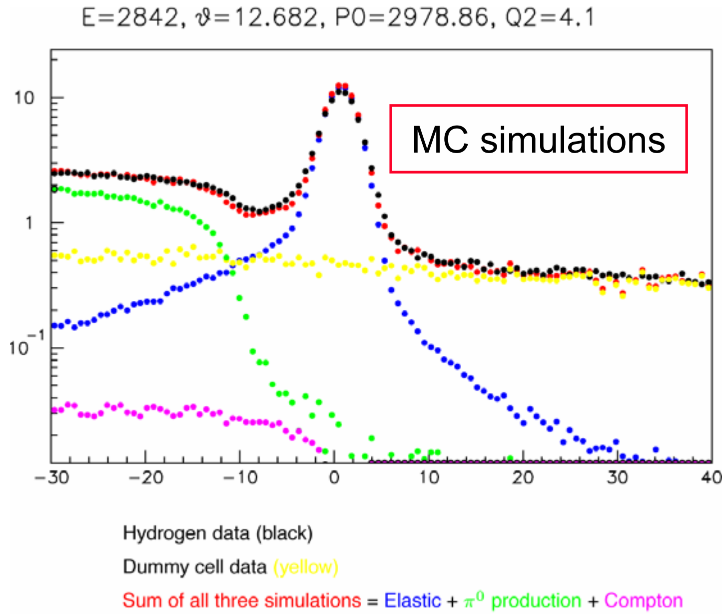
→ Investigate possible source, first by doing optimized Rosenbluth experiment

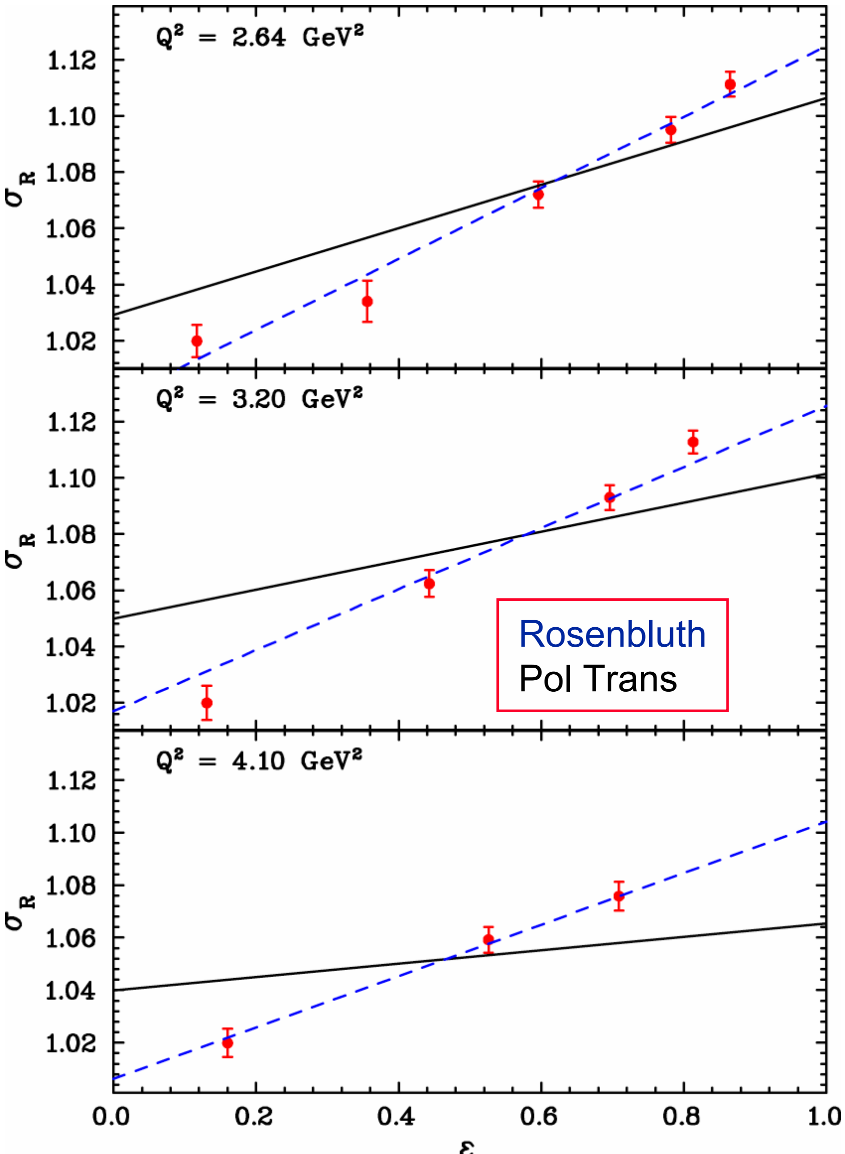


# Super-Rosenbluth (E01-001) $^1H(e,p)$

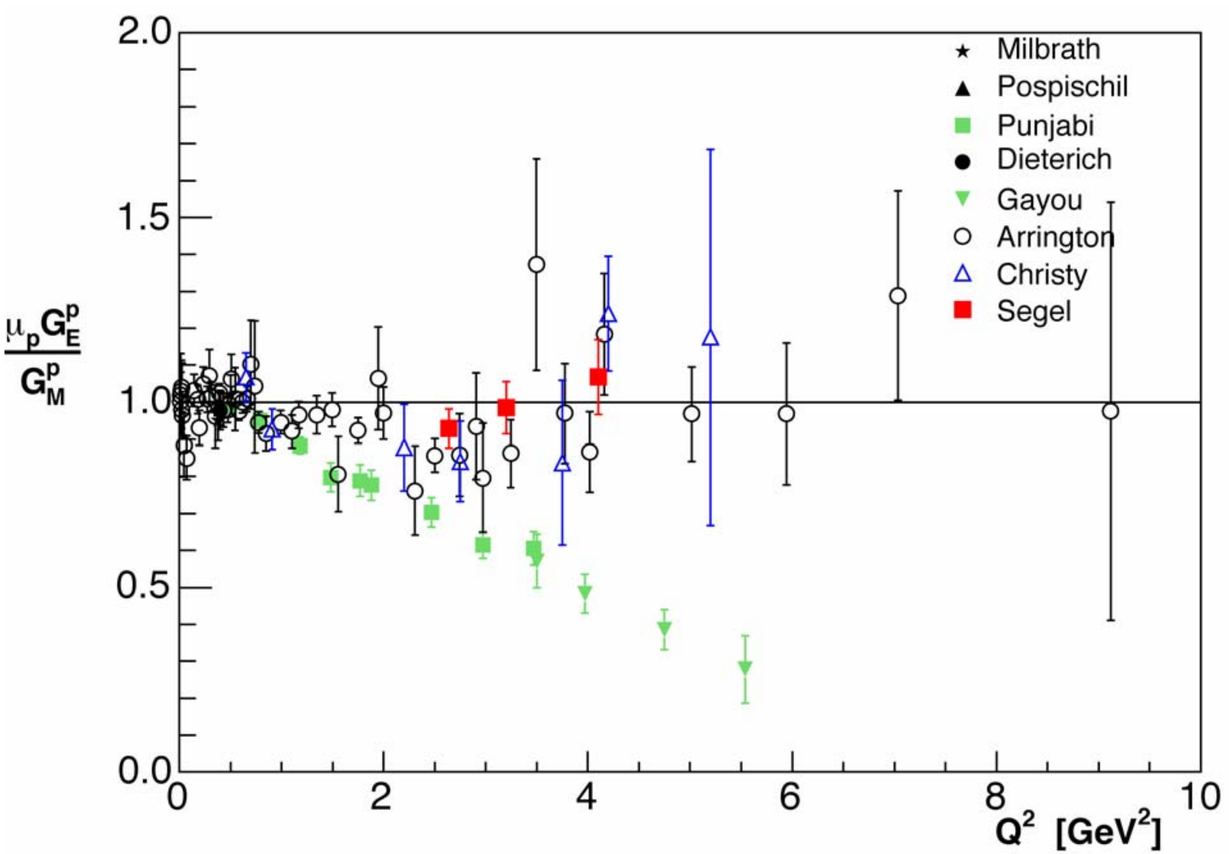
J. Arrington and R. Segel

- Detect recoil protons in HRS-L to diminish sensitivity to:
  - Particle momentum and angle
  - Data rate
- Use HRS-R as luminosity monitor
- Very careful survey
- Careful analysis of background



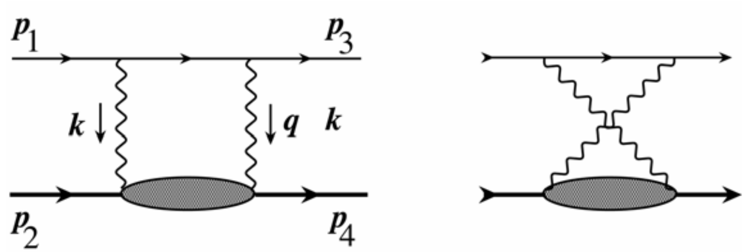
# Rosenbluth Compared to Polarization Transfer




- John Arrington performed detailed reanalysis of SLAC data
- Hall C Rosenbluth data (E94-110, Christy) in agreement with SLAC data
- No reason to doubt quality of either Rosenbluth or polarization transfer data

$\longrightarrow$ Investigate possible theoretical sources for discrepancy

# Two-photon Contributions

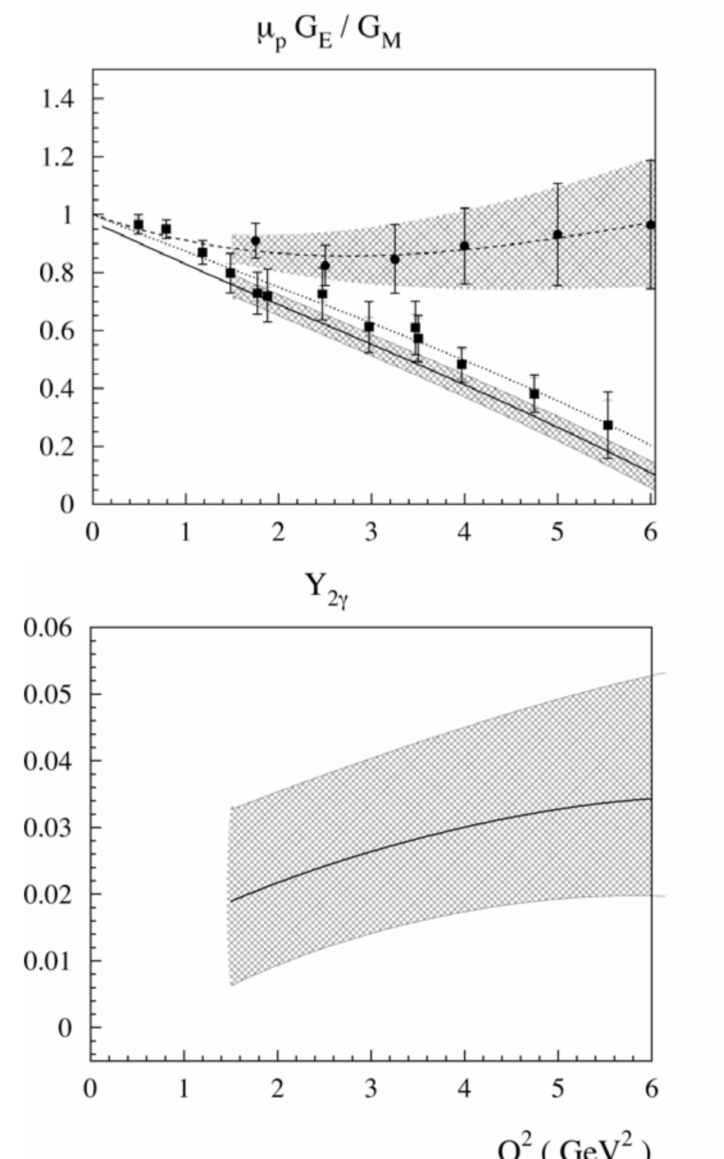



Guichon and Vanderhaeghen (PRL 91 (2003) 142303) estimated the size of two-photon effects (TPE) necessary to reconcile the Rosenbluth and polarization transfer data

$$\frac{d\sigma}{d\Omega} \propto \frac{\left|\tilde{G}_M\right|^2}{\tau}\left\{\tau + \varepsilon\frac{\left|\tilde{G}_E\right|^2}{\left|\tilde{G}_M\right|^2} + 2\varepsilon\left(\tau + \frac{\left|\tilde{G}_E\right|}{\left|\tilde{G}_M\right|}\right)Y_{2\gamma}(\nu,Q^2)\right\}$$

$$\frac{P_t}{P_l} \approx -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\left\{\frac{\left|\tilde{G}_E\right|}{\left|\tilde{G}_M\right|} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{\left|\tilde{G}_E\right|}{\left|\tilde{G}_M\right|}\right)Y_{2\gamma}(\nu,Q^2)\right\}$$

Need ~3% value for $Y_{2\gamma}$ (6% correction to $\varepsilon$-slope), independent of $Q^2$, which yields minor correction to polarization transfer



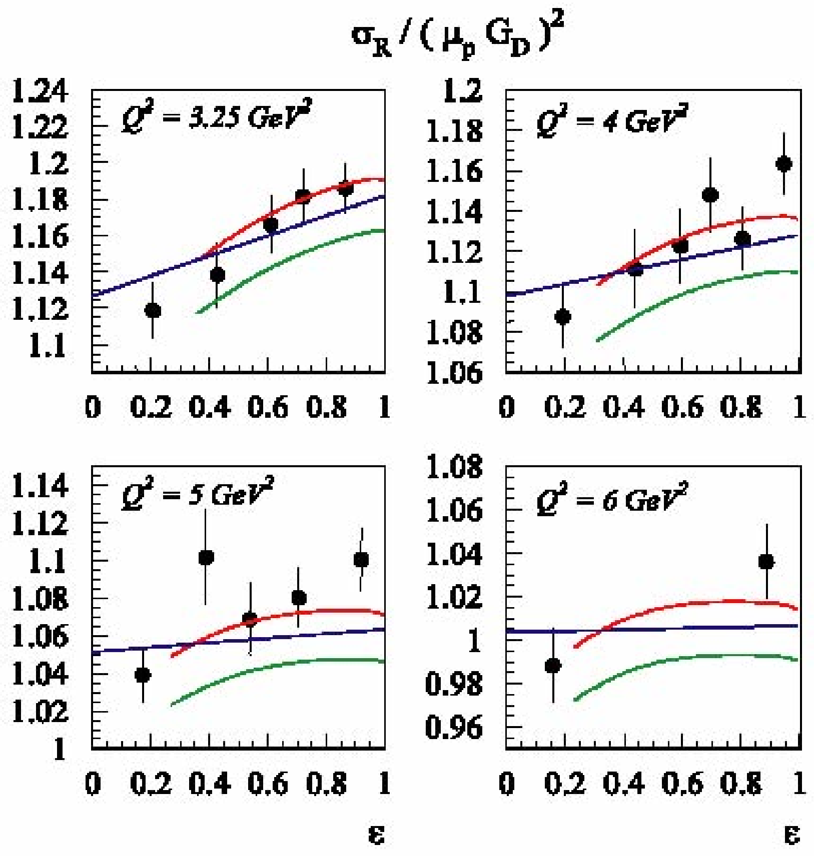
# Two-Photon Contributions (cont.)

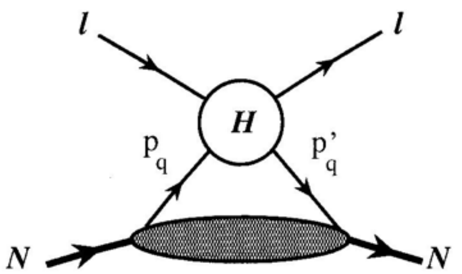
Blunden et al. have calculated elastic contribution of TPE

Resolves ~50% of discrepancy

Chen et al., hep/ph-0403058
Model schematics:

- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
- Assume factorization

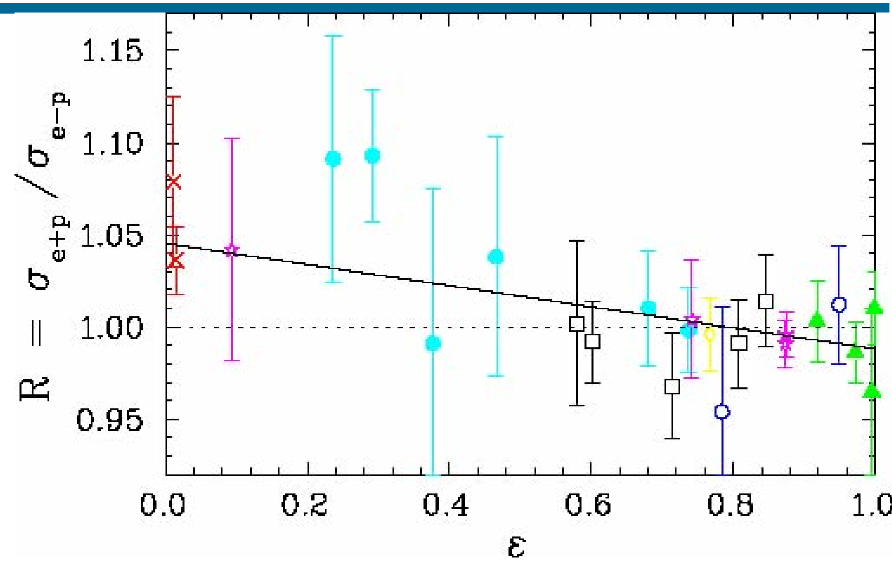


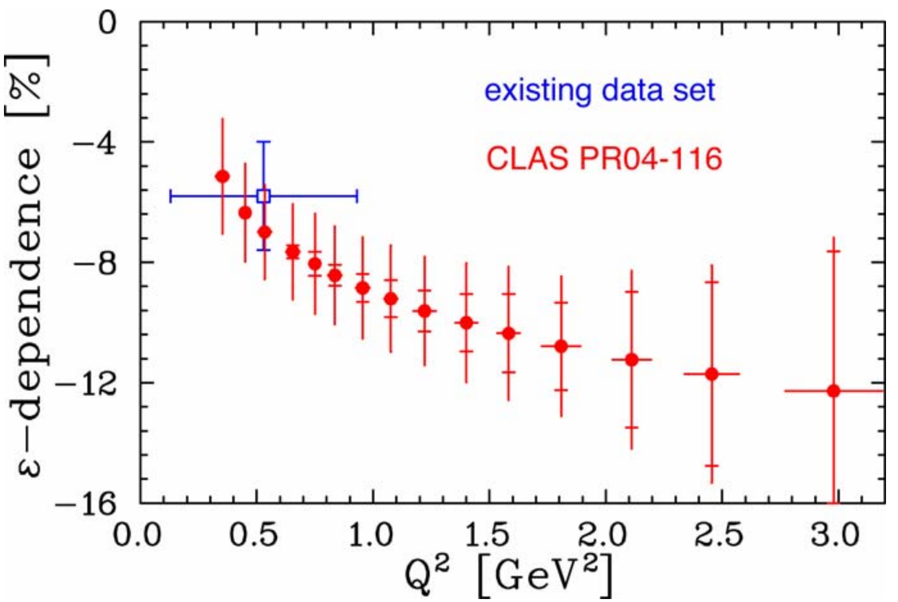

Polarization transfer
$1\gamma+2\gamma$(hard)
$1\gamma+2\gamma$(hard+soft)

# Experimental Verification of TPE contributions

Experimental verification
- non-linearity in ε-dependence
  (test of model calculations)
- transverse single-spin asymmetry (imaginary part of two-photon amplitude)
- ratio of $e^+p$ and $e^-p$ cross section (direct measurement of two-photon contributions)



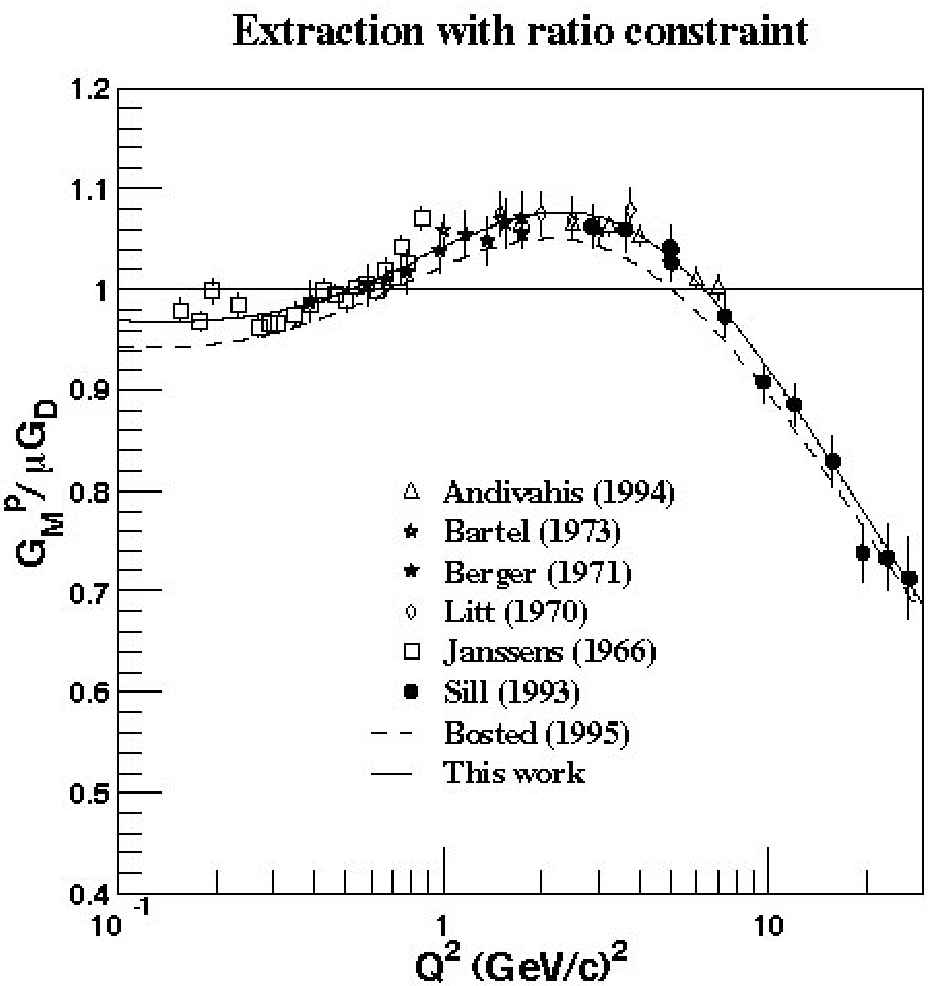

CLAS proposal PR04-116 aims at a measurement of the ε-dependence for $Q^2$-values up to 2.0 $GeV^2$



# Reanalysis of SLAC data on $G_M^p$

E. Brash *et al.*, PRC submitted, have reanalyzed SLAC data with JLab $G_E^p/G_M^p$ results as constraint, using a similar fit function as Bosted

Reanalysis results in 1.5-3% increase of $G_M^p$ data



# Theory I

→ Vector Meson Dominance
Photon couples to nucleon exchanging vector meson ($\rho,\omega,\phi$)
Adjust high-$Q^2$ behaviour to pQCD scaling
Include $2\pi$-continuum in finite width of $\rho$

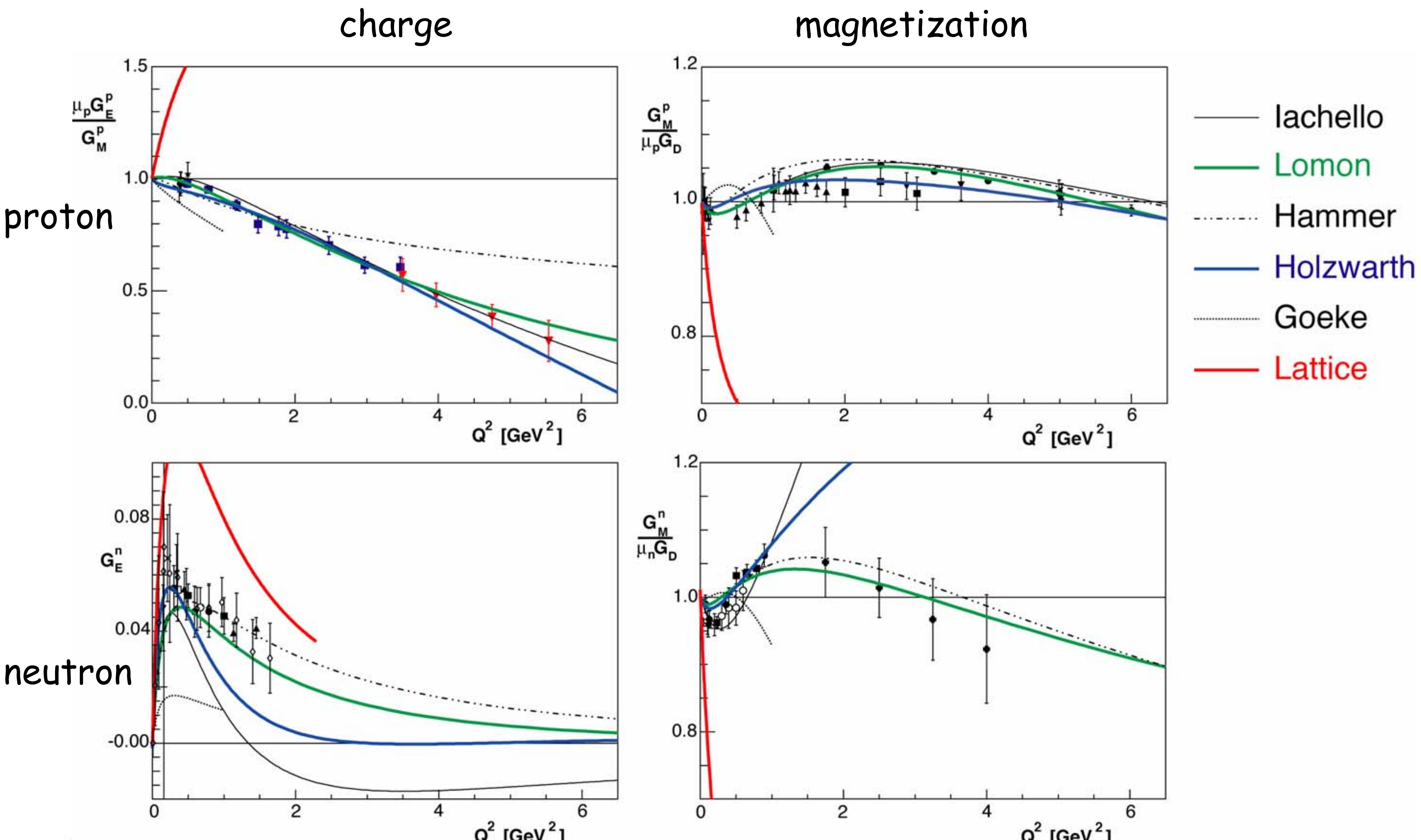
- Lomon — 3 isoscalar, isovector poles, intrinsic core FF
- Iachello — 2 isoscalar, 1 isovector pole, intrinsic core FF
- Hammer — 4 isoscalar, 3 isovector poles, no additional FF

→ Relativistic chiral soliton model

- Holzwarth — one VM in Lagrangian, boost to Breit frame
- Goeke — NJL Lagrangian, few parameters

→ Lattice QCD (Schierholz, QCDSF)
quenched approximation, box size of 1.6 fm, $m_\pi$ = 650 MeV
chiral "unquenching" and extrapolation to $m_\pi$ = 140 MeV (Adelaide)

# Vector-Meson Dominance Model



# Theory II

- Relativistic Constituent Quark Models
  Variety of q-q potentials (harmonic oscillator, hypercentral, linear)
  Non-relativistic treatment of quark dynamics, relativistic EM currents
- Miller: extension of cloudy bag model, light-front kinematics
  wave function and pion cloud adjusted to static parameters
- Cardarelli & Simula
  Isgur-Capstick oge potential, light-front kinematics
  constituent quark FF in agreement with DIS data
- Wagenbrunn & Plessas
  point-form spectator approximation
  linear confinement potential, Goldstone-boson exchange
- Giannini et al.
  gluon-gluon interaction in hypercentral model
  boost to Breit frame
- Metsch et al.
  solve Bethe-Salpeter equation, linear confinement potential

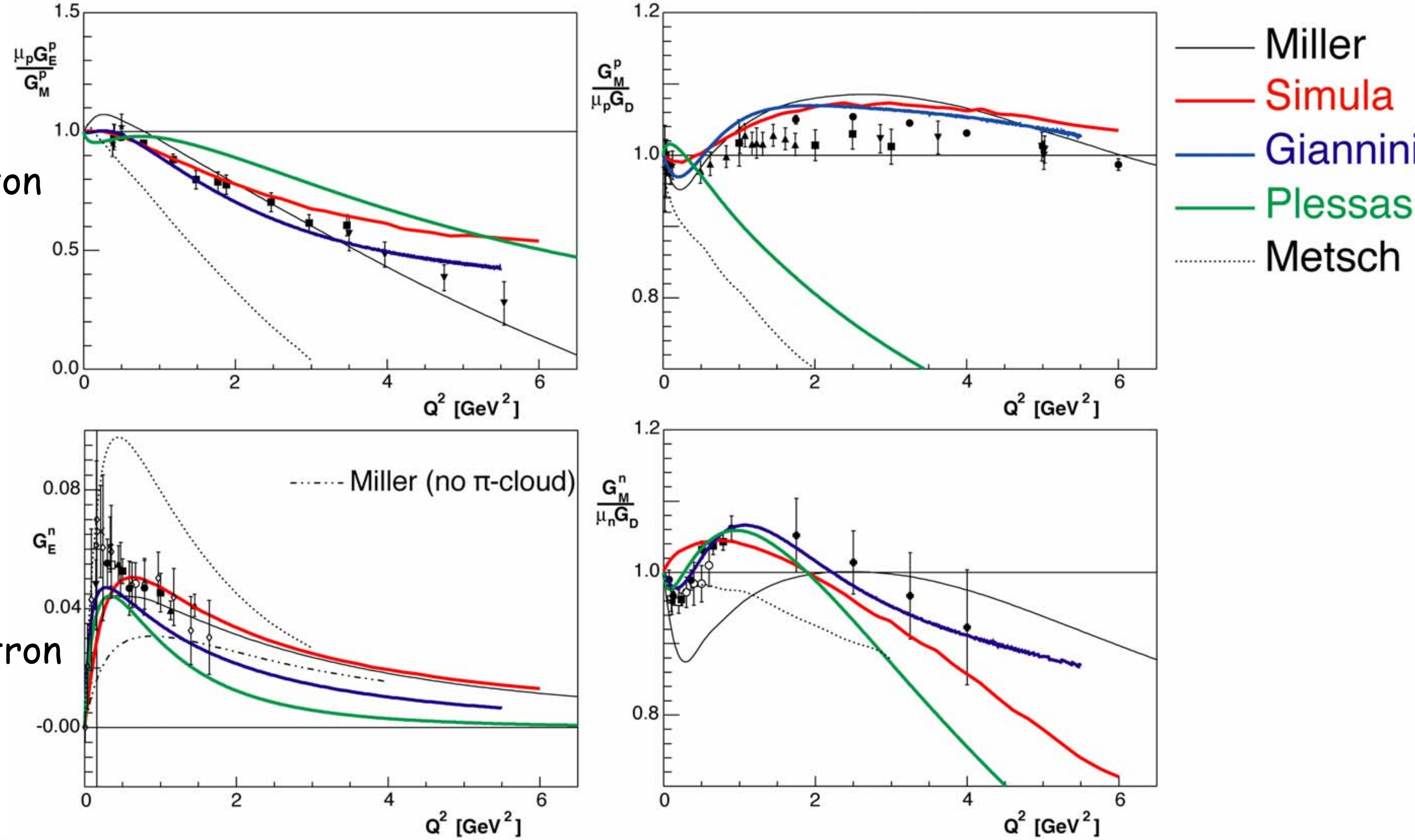
# Relativistic Constituent Quark Model

charge

magnetization

proton

neutron



# High-$Q^2$ behaviour

Basic pQCD scaling (Bjørken) predicts

$F_1 \propto 1/Q^4$ ; $F_2 \propto 1/Q^6$

$\Rightarrow$ $F_2/F_1 \propto 1/Q^2$

Data clearly do not follow this trend

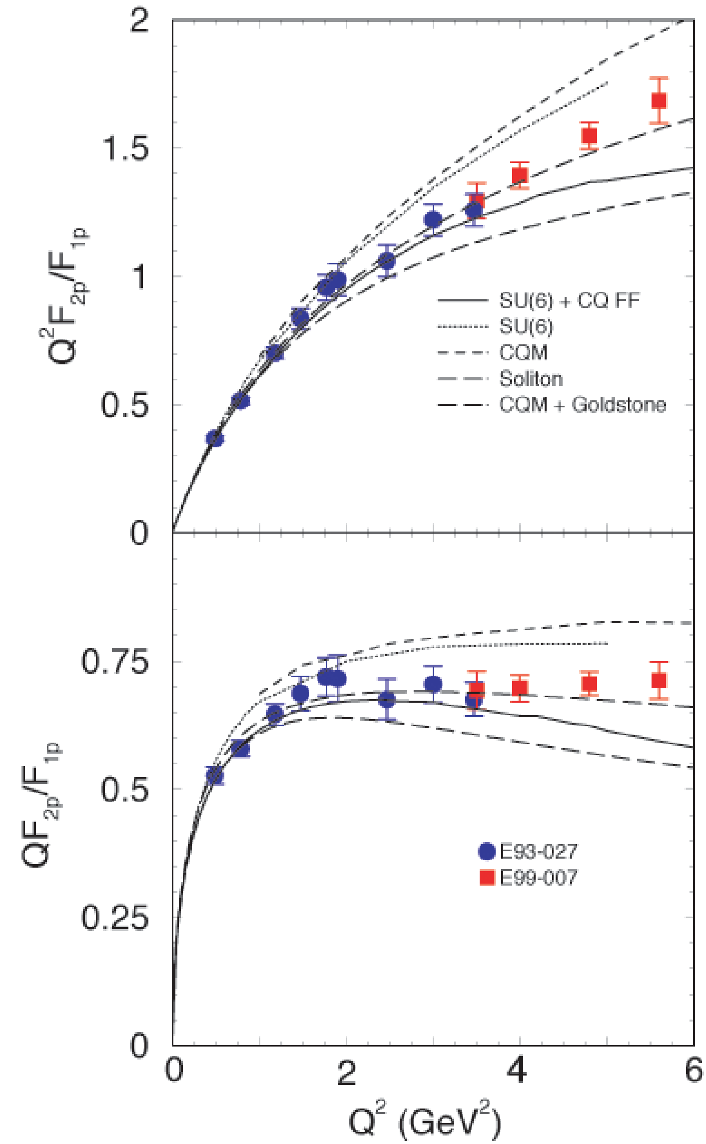
Schlumpf (1994), Miller (1996) and Ralston (2002) agree that by

- freeing the $p_T$=0 pQCD condition
- applying a (Melosh) transformation to a relativistic (light-front) system
- an orbital angular momentum component is introduced in the proton wf (giving up helicity conservation) and one obtains

  $\Rightarrow$ $F_2/F_1 \propto 1/Q$

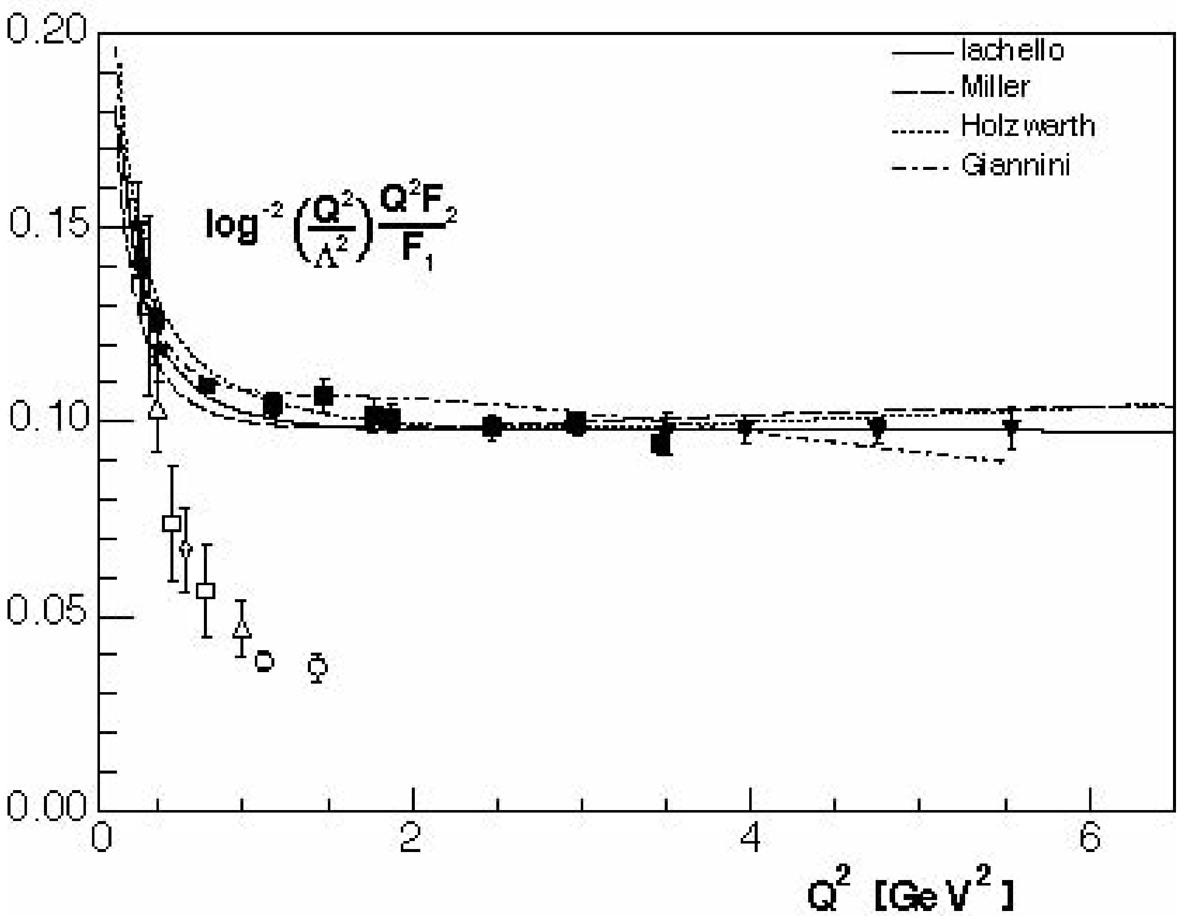
- or equivalently a linear drop off of $G_E/G_M$ with $Q^2$

Brodsky argues that in pQCD limit non-zero OAM contributes to $F_1$ and $F_2$



# High-$Q^2$ Behaviour (cont)

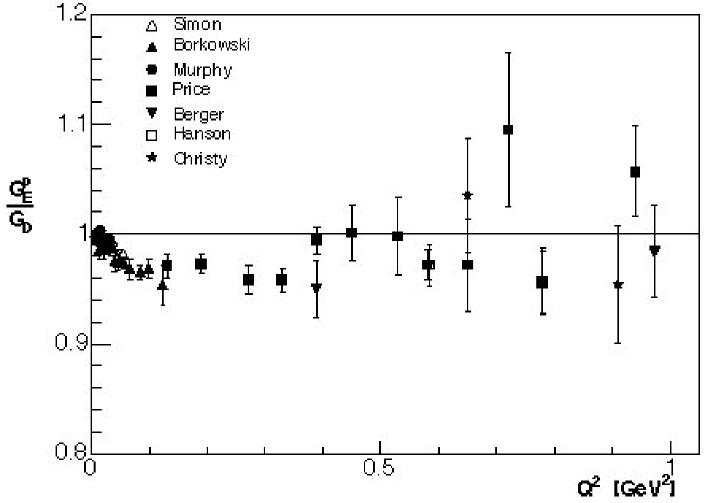
Belitsky et al. have included logarithmic corrections in pQCD limit

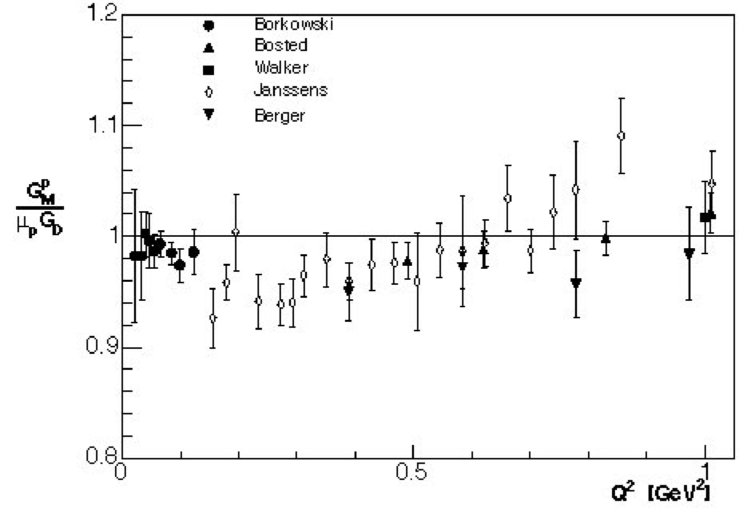


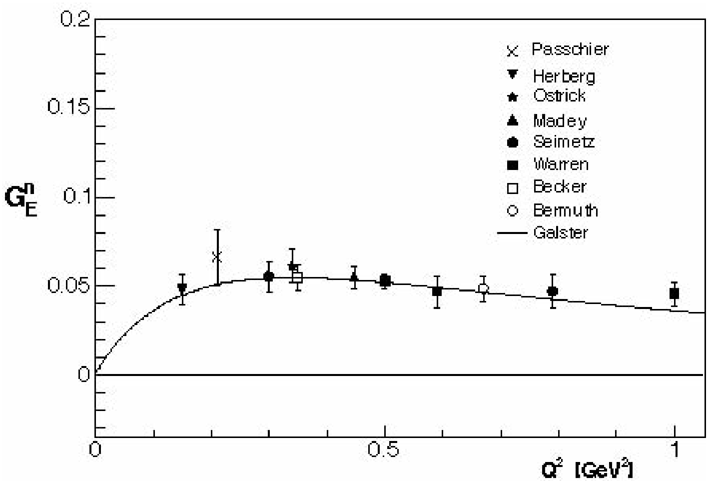

Solid: proton
open: neutron

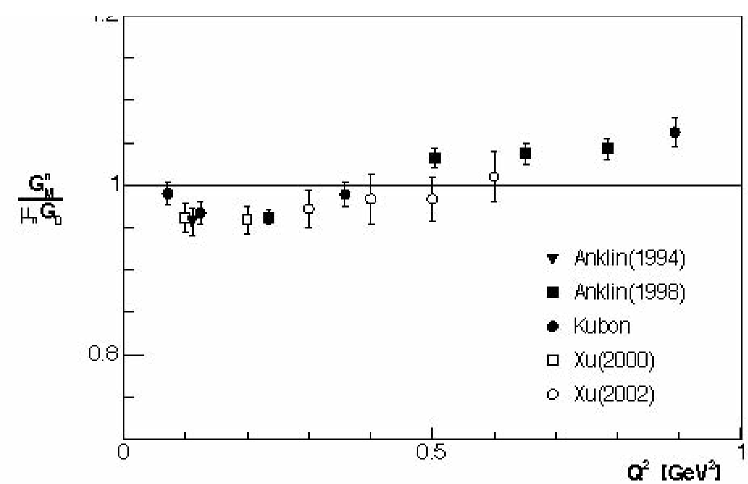
They warn that the observed scaling could very well be precocious

# Low-$Q^2$ Behaviour



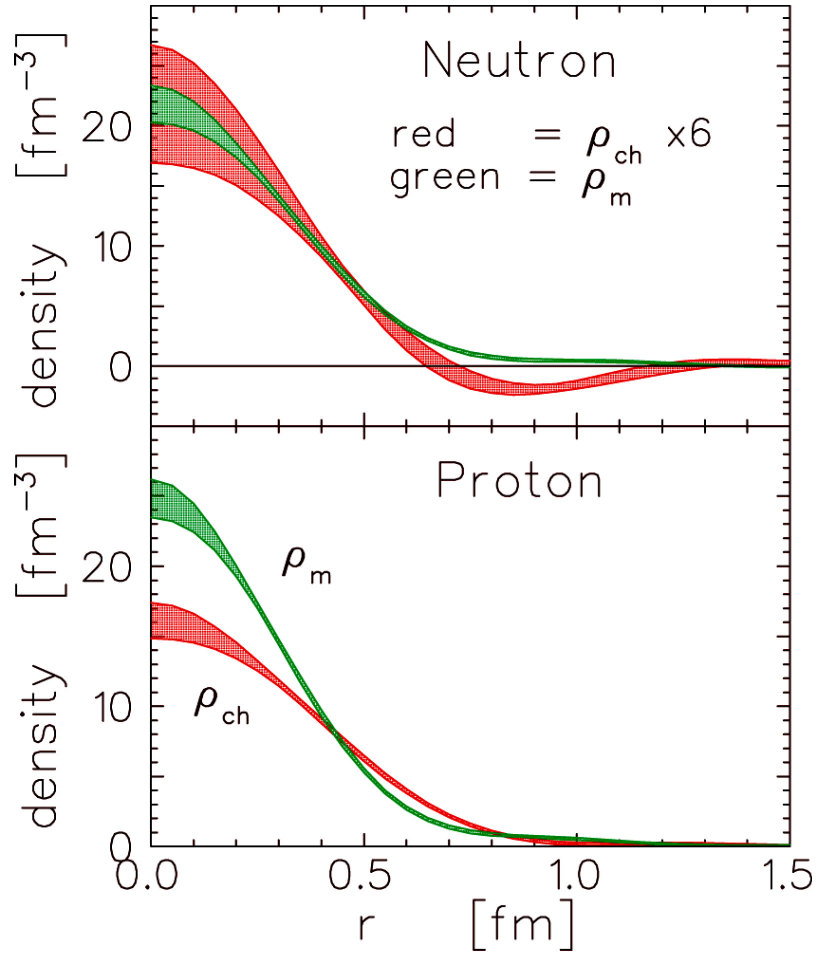

All EMFF allow shallow minimum (max for $G_E^n$) at Q ~ 0.5 GeV

# Pion Cloud

- Kelly has performed simultaneous fit to all four EMFF in coordinate space using Laguerre-Gaussian expansion and first-order approximation for Lorentz contraction of local Breit frame

$$\tilde{G}_{E,M}(k) = G_{E,M}(Q^2)(1+\tau)^2 \quad \textbf{with} \quad k^2 = \frac{Q^2}{1+\tau} \quad \textbf{and} \quad \tau = \left(\frac{Q}{2M}\right)^2$$

  - Friedrich and Walcher have performed a similar analysis using a sum of dipole FF for valence quarks but neglecting the Lorentz contraction
  - Both observe a structure in the proton and neutron densities at ~0.9 fm which they assign to a pion cloud




- Hammer et al. have extracted the pion cloud assigned to the $N\bar{N}2\pi$ component which they find to peak at ~ 0.4 fm

# Summary

- Very successful experimental program at JLab on nucleon form factors thanks to development of polarized beam (> 100 $\mu A$, > 75 %), polarized targets and polarimeters with large analyzing powers
- $G_E^n$ 3 successful experiments, precise data up to $Q^2 = 1.5$ GeV$^2$
- $G_M^n$ $Q^2 < 1$ GeV$^2$ data from $^3He(e,e')$ in Hall A
  $Q^2 < 5$ GeV$^2$ data from $^2H(e,e'n)/^2H(e,e'p)$ in CLAS
- $G_E^p$ Precise polarization-transfer data set up to $Q^2 = 5.6$ GeV$^2$
  New Rosenbluth data from Halls A and C confirm SLAC data
- Strong support from theory group on two-photon corrections, making progress towards resolving the experimental discrepancy between polarization transfer and Rosenbluth data
- Accurate data will become available at low $Q^2$ on $G_E^p$ and $G_E^n$ from BLAST
- JLab at 12 GeV will make further extensions to even higher $Q^2$ possible